

MST125

TMA 03

2018J

Covers Units 9, 10 and 11

Cut-off date 17 April 2019

You can submit this TMA either by post to your tutor or electronically as a PDF file by using the University's online TMA/EMA service.

Before starting work on it, please read the document *Student guidance for preparing and submitting TMAs*, available from the 'Assessment' area of the MST125 website.

The work that you submit should include your working as well as your final answers. Your solutions should not involve the use of Maxima, except in those parts of questions where this is explicitly required or suggested. Your solutions should not include the use of any other mathematical software. If you have a disability that makes it difficult for you to attempt any of these questions, then please contact your Student Support Team or your tutor for advice.

Your work should be written in good mathematical style, as demonstrated by the example and activity solutions in the study units. You should explain your solutions carefully, using appropriate notation and terminology, and write in sentences. As usual, you should simplify algebraic answers where possible. Five marks (referred to as good mathematical communication, or GMC, marks) on this TMA are allocated for how well you do this.

Your score out of 5 for GMC will be recorded against Question 9. You do not have to submit any work for Question 9.

Question 1 – 5 marks

You should be able to answer this question after studying Unit 9.

You are told that:

If Frosty is a cat, then Frosty has a tail.

Frosty does not have a tail.

You deduce that:

Frosty is not a cat.

Determine whether this deduction is valid, justifying your answer.

[5]

Question 2 – 10 marks

You should be able to answer this question after studying Unit 9.

- (a) Prove that the following statement is true for all real numbers x by using a sequence of equivalences:

$$x = \frac{2y}{3y-1} \text{ if and only if } y = \frac{x}{3x-2}, \text{ where } x \neq \frac{2}{3} \text{ and } y \neq \frac{1}{3}. \quad [3]$$

- (b) Prove that the following statement is true for all positive integers m and n :

$$m \text{ and } n \text{ are multiples of each other if and only if } m = n. \quad [7]$$

Question 3 – 10 marks

You should be able to answer this question after studying Unit 9.

- (a) Use mathematical induction to prove that the following statement is true:

$$n^3 - n \text{ is a multiple of 3 for all } n \geq 1. \quad [9]$$

- (b) Show that the following statement is false:

$$n^4 - n \text{ is a multiple of 4 for all } n \geq 1. \quad [1]$$

Question 4 – 10 marks

You should be able to answer this question after studying Unit 9.

- (a) Use proof by contradiction to show that there is no positive real number x such that

$$\frac{4x^2 - 6x + 9}{3x} < 2. \quad [4]$$

- (b) Use proof by contraposition to prove that the following statement is true for all positive integers n :

$$\text{If } n^2 + 3n \text{ is a multiple of 3, then } n \text{ is a multiple of 3.} \quad [6]$$

Question 5 – 5 marks

You should be able to answer this question after studying Unit 10.

In a race, a homing pigeon accelerates from rest with constant acceleration for 40 s to reach a cruising speed of 22 m s^{-1} .

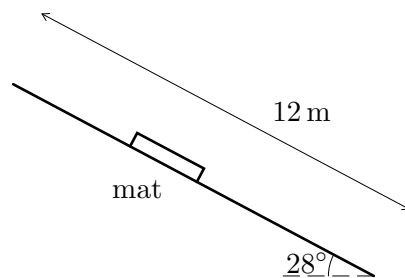
Assume that the pigeon flies in a straight line. Give numerical answers to two significant figures.

- (a) What is the acceleration of the pigeon? [2]
- (b) Over what distance does the pigeon accelerate? [2]
- (c) Draw a graph of the velocity v (in metres per second) of the pigeon against the time t (in seconds), from the time at which the pigeon sets off until 100 s into the race. [1]

Question 6 – 15 marks

You should be able to answer this question after studying Unit 10.

A flat straight grass slide makes an angle of 28° with the horizontal and is 12 m long. A mat starts sliding from rest at the top of the slide. The mat has constant acceleration and it reaches a speed of 2.5 m s^{-1} at the bottom of the slide. Let the mass of the mat be m (in kg) and the coefficient of sliding friction between the mat and the slide be μ .



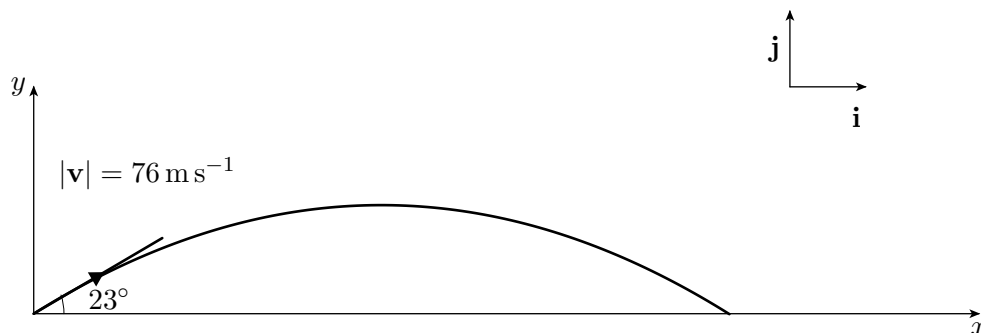
Model the mat as a particle and take the magnitude of the acceleration due to gravity to be $g = 9.8 \text{ m s}^{-2}$.

- (a) Show that the acceleration of the mat is 0.26 m s^{-2} to two significant figures. [2]
- (b) Draw a force diagram showing the forces acting on the mat when it is sliding down the slide. Define the symbols you use to denote the forces. Choose appropriate directions for the Cartesian unit vectors $\mathbf{i}$ and $\mathbf{j}$, and mark these vectors on your diagram. [4]
- (c) Find the coefficient of sliding friction between the mat and the slide to two significant figures.
(Note: To answer this part of the question, you should use Steps 5 to 8 and Steps 10 to 11 of the strategy 'To solve a dynamics problem involving several forces', which is described on page 110 of the Handbook, and also the *full calculator accuracy* value of the acceleration from part (a).) [9]

Question 7 – 15 marks

You should be able to answer this question after studying Unit 10.

A golf ball rests on horizontal ground. It is hit at an angle of 23° above the horizontal at a speed of 76 m s^{-1} . Set up coordinate axes at the point where the golf ball is hit, as shown below.



Let $\mathbf{i}$ and $\mathbf{j}$ be the Cartesian unit vectors in the positive directions of the x - and y -axes respectively.

Assume that the ball can be modelled as a particle and that the only force acting on it is its weight. Take the magnitude of the acceleration due to gravity to be 9.8 m s^{-2} . Give numerical answers to two significant figures.

- (a) Show that the position $\mathbf{r}$ (in metres) of the ball t seconds after it has been hit is given by

$$\mathbf{r} = 76t \cos 23^\circ \mathbf{i} + (76t \sin 23^\circ - \tfrac{1}{2}gt^2)\mathbf{j}. \quad [8]$$

- (b) Find the time (in seconds) that it takes for the ball to reach the ground. [5]
 (c) Determine the horizontal distance (in metres) between the point where the ball is hit and the point where it first hits the ground. [2]

Question 8 – 25 marks

You should be able to answer this question after studying Unit 11.

Let $\mathbf{A} = \begin{pmatrix} 10 & 40 \\ 3 & 8 \end{pmatrix}$.

- (a) Find the eigenvalues of $\mathbf{A}$, and find an eigenvector corresponding to each eigenvalue. [8]
 (b) Hence express $\mathbf{A}$ in the form $\mathbf{PDP}^{-1}$, clearly stating the matrices $\mathbf{P}$, $\mathbf{P}^{-1}$ and $\mathbf{D}$. [4]
 (c) Use your answer to part (b) to calculate $\mathbf{A}^4$. [4]
 (d) Use Maxima to check your answers to parts (a) and (c). Include a printout or screenshot of the Maxima output in your answer, and explain carefully how to interpret it. [5]
 (e) Use your answer to part (a) to find the general solution of the following system of linear differential equations:

$$\begin{aligned} \dot{x} &= 10x + 40y, \\ \dot{y} &= 3x + 8y. \end{aligned} \quad [4]$$

Question 9 – 5 marks

Five marks on this assignment are allocated for good mathematical communication in Questions 1 to 8.

You do not have to submit any extra work for Question 9, but you are advised to check through your assignment carefully, making sure that you have explained your working clearly, used notation correctly, written in sentences and rounded answers as requested.

[5]
